Recreational Mathematics Colloquium VII–G4G Europe

BOOK OF ABSTRACTS

Expolab–Centro Ciência Viva, Lagoa
São Miguel, Azores Islands, Portugal

January 27th–29th, 2023

Organization:

Ludus Association

Support:

Câmara Municipal da Lagoa
Centro de Matemática Aplicada à Previsão e Decisão Económica
Centro Interuniversitário de História da Ciência e Tecnologia
Expolab–Centro de Ciência Viva
Faculdade de Ciências e Tecnologia–Universidade dos Açores
Gathering 4 Gardner Foundation
Governo dos Açores
Núcleo Interdisciplinar da Criança e do Adolescente
Sociedade Afonso Chaves
Sociedade Portuguesa de Matemática

Organizing Committee:

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Ana Paula de Ornelas Garrão, Portugal
Anabela Teixeira, Portugal
Jorge Nuno Silva, Portugal
Margarida de Jesus Silva Raposo, Portugal
Ricardo Teixeira, Portugal
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Jorge Buescu, Portugal
Jorge Nuno Silva, Portugal
Pedro Freitas, Portugal
Richard Nowakowski, Canada
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Tony Mann, UK

Foreword

The seventh Recreational Mathematics Colloquium was scheduled to take place in 2019. Unfortunately, we were all visited by an uninvited virus and had to postpone it. Now, we are back! It is with great emotion that we welcome you all to our mathematical pow-wow!

Ludus Association is determined to keep promoting these Colloquia for many years to come! This tradition, which started in 2009, will carry on.

We are back in the beautiful island of São Miguel (RMC III happened here), so we can enjoy this fantastic environment. Besides, the organization of this event relies on the competence and good will of its local team.

RMC-VII (G4G-Europe) will be, we hope, a success like the previous editions. Unfortunately, we will miss the presence of some old compagnons de route, but we know that this is a consequence of the natural laws of life. Let's welcome the newcomers, many of which are young and will stay with us for long.

Ludus Association counted with the kind support of Governo dos Açores, ExpoLab, Câmara Municipal da Lagoa, FCT-Universidade dos Açores, NICA, Ciência Viva, G4G, SPM, FCT, CMAF-IO (UIDB/04561/2020), CIUHCT, and CEMAPRE.

We are proud to organize the seventh colloquium in the series, the fourth integrated in the Gathering for Gardner movement. There will be two natural outlets for written versions of presented works. The Proceedings of the RMC VII will be published and made available through the Amazon platform, as usual. Some of the papers will be selected to appear in the Recreational Mathematics Magazine (`https://sciendo.com/journal/RMM`). Please send your submissions to ludus@ludicum.org until July 31st, 2023.

You can get more information at `http://ludicum.org/ev/rm/21`

The organization

Program

Friday, 27th January

Buses at 8:30 & 9:00, Hotel Canadiano, Ponta Delgada ⟶ Expolab, Lagoa

9:00 **Registration**

10:00 **Opening Ceremony**

Chair: Robert Vallin

10:15 **Yupana, The Inca Abacus**
Jorge Nuno Silva, Ludus

10:30 **Maths for time travellers, briefing 17**
Adam Atkinson

10:45 **Taking things further?**
Colin Wright, Solipsys Ltd

11:00 Coffee-Break

11:30 **Some new things from a deck shuffle**
Robert Vallin, Lamar University

11:45 **Dice battles and conquering probabilities**
Flavia Sancier-Barbosa, Colorado College

12:00 **Mental Calculus by Nikolai Petrovich Bogdanov-Belsky**
Alda Carvalho, ISEL-IPL & CEMAPRE/REM-University of Lisbon

12:15 Break for lunch

Chair: José Carlos Santos

14:00 **Arithmetic for game-playing robots**
Richard Nowakowski, Dalhousie University

14:15 **Stacking placement games**
Svenja Huntemann, Concordia University of Edmonton

14:30 **Bottled problems**
Joaquim Eurico Nogueira, FCT-UNL

14:45 **Self-tiling sets of polygones and polycubes**
Sabine Segre & Einav Aizikovitsh-Udi, Achva Academic College

15:00 **Some notes on Escher's regular division of the plane**
Carlos Santos, Center for Mathematics and Applications (NovaMath), FCT NOVA

15:15 Coffee-Break

15:45 **How hard is to tell whether a figure tiles? The quest for figures with large Heesch numbers**
Bojan Bašić, University of Novi Sad

16:00 **Math coloring book**
Dirk Huylebrouk

16:15 **The recreational problems of the 1519 Treatise of Gaspar Nicolas**
Pedro Freitas, FCUL, University of Lisbon

17:00 **Welcome cocktail**

Buses at 18:00 & 18:30, Expolab, Lagoa → Hotel Canadiano, Ponta Delgada

Saturday, 28th January

Buses at 8:30 & 9:00, Hotel Canadiano, Ponta Delgada → Expolab, Lagoa

Chair: Carlota Simões

9:30 **Probabilistic play past primal proofs**
Jorge Buescu, FCUL & CMAFCIO

9:45 **Squares with large digit average**
Silvia Heubach, California State University

10:00 **Hunting the algorithm (Part 1)**
Sergio Belmonte, MMACA

10:15 **Magic squares as an introduction to mathematical proofs**
José Carlos Santos, University of Oporto

10:30 **Meer's + Maths = Fermat, Kraïtchik (1883-1957) and his contributions to recreational mathematics**
Lisa Rougetet, Centre François Viète - Université de Bretagne Occidentale
Tiago Hirth, ULisboa, CIUHCT, Ludus

10:45 **The mathetics of tilt**
David Richeson, Dickinson College

11:00 Coffee-Break

11:30 **A discrete look at Hilbert's incidence axioms**
Milica Maksimovič, University of Novi Sad

11:45 **COVIDiary of mathematicians**
Aleksandra Ravas

12:00 **Reflections on G4G and the legacy of Martin Gardner**
Mark Mitton, Mark Mitton, Inc.

12:15 Break for lunch

Buses at 14:00 & 14:30, Expolab, Lagoa → Hotel Canadiano, Ponta Delgada

Conference dinner, 20:00-22:30, *Hotel Marina Atlantico*

Address: Av. João Bosco Mota Amaral, 1, 9500-767 Ponta Delgada

Sunday, 29th January

Buses at 8:30 & 9:00, Hotel Canadiano, Ponta Delgada → Expolab, Lagoa

Chair: Colin Wright

9:30 **Digital learning games for mathematics and computer science education**
Muhammad Akram, National College of Business Administration and Economics

9:45 **The "mathematical recreations kit" or the use of historical recreations to teach maths**
Lisa Rougetet, Centre François Viète - Université de Bretagne Occidentale

10:00 **Bidding combinatorial games**
Urban Larsson, IIT Bombay

10:15 **Astronomy in literature and poetry**
Carlota Simões, University of Coimbra

10:30 **Domino antimagic squares**
Alison Marr, Southwestern University

10:45 **Ludus Regularis–A game from the 10th century**
Carla Cardoso, University of Lisbon

11:00 Coffee-Break

11:30 **Hunting the algorithm (Part2)**
Guido Ramellini, MMACA

11:45 **FUNDAPROMAT: spreading the joy of mathematics**
Jeanette Shakalli, Panamanian Foundation for the Promotion of Mathematics

12:00 **Calendars: proposals and ideas**
Fernando Blasco

12:15 Break for lunch

Buses at 14:00 & 14:30, Expolab, Lagoa → Hotel Canadiano, Ponta Delgada

15:30 *Biblioteca Pública de Ponta Delgada* (Public Library)
Address: Av. Gaspar Frutuoso, 9500-054 Ponta Delgada

Matinée Magic Show
Adrien Lochon | France
Colin Wright | Australia
Katerina L'dokova | Belarus
Sergio Belmonte | Spain
Robert Vallin | USA
Circo Matemático | Portugal

16:45 **Launching of Jorge Buescu's book *Amor, Matemática e Outros Portentos***
Carlota Simões, University of Coimbra

17:00 **Closing Ceremony**

Abstracts

Yupana, The Inca Abacus
Jorge Nuno Silva, Ludus

In *The First New Chronicle and Good Government—On the History of the World and the Incas up to 1615*, Felipe Guaman Poma de Ayala shows an image of a man holding a Quipu—a device designed to keep records—having by him what looks like a calculation table to be operated with pebbles: a *Yupana*. It was, all believe, meant to help with arithmetic calculations, but there are several opinions about the way it was meant to work. We'll give our opinion on the subject and go through the many possibilities of this fantastic gadget.

Maths for time travellers, briefing 17
Adam Atkinson

Ludus, like many organizations, has a time travel division. Clothing, language, sword fighting and dinosaur hunting are useful skills, but sometimes when you travel in time you may need to do maths while people are watching you.

Taking things further?
Colin Wright, Solipsys Ltd

There are many classic puzzles where people solve the puzzle as set, and then stop. In this talk we look at a few puzzles and ask . . . what next? Can we take this further? And can we take things "too" far?

Some new things from a deck shuffle
Robert Vallin, Lamar University

The mathematical study of shuffling is nothing new. This talk concentrates on the so-called Australian Under-Down Shuffle. We introduce the shuffle and answer a question from E. Behrends on a variation of the shuffle. This results in a new sequence for the Online Encyclopedia of Integer Sequences. Then we dissect this sequence from a couple of points of view. Finally, we look at fixed cards in the shuffle and apply this to a card trick.

Dice battles and conquering probabilities
Flavia Sancier-Barbosa, Colorado College

Inspired by the board game RISK, where the roll of dice determines the outcome of a battle, we explore how the probabilities of winning battles

change when the number of attacking and defending dice change. Most of the mathematical literature on RISK does not arrive at simple formulas for conquering probabilities. In this talk, I will present how formulas can be derived for these probabilities and will share insights into how the game dynamics change as the number of dice changes.

Mental Calculus by Nikolai Petrovich Bogdanov-Belsky
Alda Carvalho, ISEL-IPL & CEMAPRE/REM-ULisboa

There are many artistic objects, such as paintings or movies, showing details with some kind of technical content (CHESS positions, math expressions, scientific facts, etc). Unfortunately, most of times, these contents are totally absurd, if not completely wrong or false. This is an unforgivable lack of care. If the artist is careful with color, contrast, planes, etc., why is he not careful with the plausibility and meaning of these details? And the problem is not just sloppiness; even more serious is missing an opportunity. This problem does not occur in the painting Mental Calculus (1895), by Nikolai Petrovich Bogdanov-Belsky (1868-1945). The main purpose of this work is to discuss the math expression in that painting.
(Joint work with Carlos Pereira dos Santos, Center for Mathematics and Applications (NovaMath), FCT NOVA).

Arithmetic for game-playing robots
Richard Nowakowski, Dalhousie University

Robots playing simultaneous games against humans have fast reflexes, fast enough to see what move the human is making. I'll explore some of the arithmetic that arises from such games. For example, we have $\frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 0$ and $\frac{-1}{2} = 0$, but $\frac{-1}{2} + \frac{-1}{2} = -1$, for some definition of equality.
(Joint work with Melissa Huggan, Vancouver Island University.)

Stacking placement games
Svenja Huntemann, Concordia University of Edmonton

Ultimate Tic-Tac-Toe is played on a 3×3 grid with each space subdivided into another 3×3 grid. Whenever a player wins a subgrid using TIC-TAC-TOE rules, they place their piece on the larger grid with the goal being to win the latter. Motivated by ULTIMATE TIC-TAC-TOE, we looked at how a similar "stacking" operation could be defined for games such as SNORT and COL where the placement of pieces depends on the distance to previously placed pieces only and the last player to place a piece wins.

Bottled problems
Joaquim Eurico Nogueira, FCT-UNL

The collection of problems "Propositiones ad acuendos juvenes" that Alcuin of York proposed to the emperor Charlemagne so that he could "delight with them" is today very-well known; although it was created to train the pupils of the palatine school, it has passed into history as one of the first collections of recreational mathematical problems. In this paper, I refer to some problems in that work, involving the filling of bottles with liquid, and some of their modern developments.

Self-tiling sets of polygones and polycubes
Sabine Segre, Achva Academic College
Einav Aizikovitsh-Udi, Achva Academic College

At G4G14 we showed that the classic tangram puzzle is a set of self-tiling tiles with more than one scaling-factor. According to Martin Gardner, the Soma cube, which was invented by Piet Hein, is the most successful attempt to create a three-dimensional analogue of tangram. The Soma cube is made of 7 polycubes, where a polycube is made by joining equal cubes face to face. We will investigate if there are self-tiling sets of tiles among the pieces of the Soma cube or among other sets of polycubes.

Some notes on Escher's regular division of the plane
Carlos Santos, Center for Mathematics and Applications (NovaMath), FCT NOVA

In this work, we discuss the geometry behind Escher's tessellations, transition devices, and an interesting interaction between Escher and the British mathematical physicist and philosopher Roger Penrose.

How hard is to tell whether a figure tiles? The quest for figures with large Heesch numbers
Bojan Bašić, University of Novi Sad

Deciding if a given figure T tiles the plane can be hard. The so-called Heesch number ranks figures by their ability to "advance" toward a tiling; it is the maximal nonnegative integer n such that T can be surrounded by its copies n times. It is unknown if there exists the largest finite Heesch number. Until recently, the "record-holder" was a figure with Heesch number 5. In the talk we break this two-decade-old record, by presenting a construction of a figure with Heesch number 6.

Math coloring book
Dirk Huylebrouk

The author published a "Mathematics Coloring Book", suggesting classic children's coloring books and adult's mandalas collections could include mathematical drawings too. This may even have a "collateral advantage", as students may get back to "learning by doing". They would be in good company: Leonardo da Vinci also colored mathematically. The talk will mainly consist of "hands-on" activities, so that all participants will get in touch with the colorful world of mathematics.

The recreational problems of the 1519 Treatise of Gaspar Nicolas
Pedro Freitas, ULisboa

The book "Tratado da prática de aritmética" (Treatise on the practice of arithmetic) by Gaspar Nicolas (1519), was the first book on mathematics to be published in Portuguese. It was recently reprinted, with notes, in the "Portuguese Culture" series of the Gulbenkian foundation. Having mostly exercises pertaining to commerce, it also contains many recreational problems. In this talk, we will present a selection of these problems.

Probabilistic play past primal proofs
Jorge Buescu, University of Lisbon

Prime numbers exhibit a mysterious and delicate mix of regular and random behavior. We will highlight several instances where correct statements about primes arise from apparently handwaving, probabilistic arguments. The Riemann zeta function, of course, takes centre stage in the emergence of order emerges from chaos.

Squares with large digit average
Silvia Heubach, California State University

Inspired by a question on squares with at most two different digits, we investigate which squares have a high average of their digits. We will provide two infinite sequences whose digit averages can be made as close to 8.25 as desired. Also, we will give a probabilistic argument that there are infinitely many squares with digit average between 8.25 and 8.3. We challenge the audience to find examples of such squares. The current record is 8.275 for the square of a 20-digit number -join the hunt!
(Joint work with Matthieu Dufour, University of Quebec).

Hunting the algorithm (Part 1)
Sergio Belmonte, MMACA

Based on a simple challenge from the MMACA, three members of the museum set out to "make a point" of the challenge. From here we were looking for a pattern or an algorithm to generalise this beautiful problem, as well as enriching the challenge more and more, turning it upside down. You can never be too adult to play like a child.

Magic squares as an introduction to mathematical proofs
José Carlos Santos, University of Oporto

In this talk, you shall view how the study of magic squares is a good introduction to understanding and making proofs.

Meer's + Maths = Fermat, Kraïtchik (1883-1957) and his contributions to recreational mathematics
Lisa Rougetet, Centre François Viète–Université de Bretagne Occidentale
Tiago Hirth, ULisboa, CIUHCT, Ludus

Maurice Kraïtchik (1882-1957) is well-renowned among those interested in recreational mathematics and, in a lesser degree in computation and number theory. His book La mathématique des jeux, first published in 1930, has widely been published, reprinted several times, and translated. In this talk, we will present a short biography of Maurice Kraïtchik as well as explore how he promoted and contributed to recreational mathematics and why his time is known as the golden age of cryptarithms.

The mathetics of tilt
David Richeson, Dickinson College

Ludus Regularis Seu Clericalis, the Clergy Game, is a board game with dice, invented in the 10th century by Bishop Wibold of Cambrai (France), with the goal of discouraging the involvement of clerics in gambling, and make them experience the pleasures of the practice of a game with a virtuous theme.

A discreete look at Hilbert's incidence axioms
Milica Maksimović, University of Novi Sad

It is curious (and more-or-less well-known) that 4 points, 6 lines and 4 planes are everything one needs in order to fulfill all Hilbert's axioms of incidence (which is the first axiom group in the foundations of the Euclidean

geometry as we know it). However, finite configurations with more than 4 points are very rarely mentioned, if at all. We present some classes of such configurations with n points for a given n, and further, for n up to 12, calculate the exact number of non-isomorphic ones.
(Joint work with Kristina Ago, Bojan Bašič and Milica Šobot.)

COVIDiary of mathematicians
Aleksandra Ravas

This is a book that was created quite by chance during the COVID virus pandemic by mathematicians from all around the world. Its authors, Tiago Hirth, Guido Ramellini, James Tanton, Jovan Kneževič, Kiran Bacche, Sergio Belmonte, Tijana Markovic, and Aleksandra Ravas, well known for their genuine curiosity and desire to acquire knowledge and share what they learn, are united by things we all have in common: the quest for the meaning of life and the search for the way to live in difficult times.

Reflections on G4G and the Legacy of Martin Gardner
Mark Mitton, Mark Mitton, Inc.

Magician Mark Mitton reflects on G4G and how it surprisingly led to his friendship with John Conway, being a teacher's assistant for Manjul Bhargava, and working with Ernő Rubik on a festival in Athens for the general public.

Digital learning games for mathematics and computer science education
Muhammad Akram, National College of Business Administration and Economics

Especially digital game-based learning (DGBL) is considered an effective educational tool for improving education in classrooms of the future. Yet, learning is a complex psychological phenomenon and the effectiveness of digital games for learning cannot be taken for granted.

The "mathematical recreations kit" or the use of historical recreations to teach maths
Lisa Rougetet, Centre François Viète–Université de Bretagne Occidentale

In 2019, I presented Ozanam Project supported by Plaisir Maths, a French association devoted to the popularization of mathematics through innovative and playful experiences. The 2023-year will see the publication of a "mathematical recreations kit" intended for pupils between 9 and 11 years old to learn mathematics through manipulation and fun, based on historical recreations. In this talk, I will present few mathematical recreations, how

they were selected and constructed for a use in class.

Bidding Combinatorial Games
Urban Larsson, IIT Bombay

Two player combinatorial games such as CHESS, HEX and GO, are usually played by taking turns. In this presentation we explore some simple game forms in a bidding generalization of alternating play. The winning convention is normal play, that is "last move wins", or more precisely "who cannot move loses". Initially we were surprised to see that those standard expressions of the same winning convention, may not be equal in bidding play. We will present an elegant solution.

Astronomy in literature and poetry
Carlota Simões, University of Coimbra

The epic poem "Os Lusíadas", by Luís de Camões (1572), is considered the most important work of Portuguese-language literature. It is well known that Camões had a clear knowledge of the principles of astronomy, as it was professed in his time. But does this accuracy apply to other authors, at that time or today? In this talk we take excerpts with references to astronomy from several authors, and we analyze their "truth" from the point of view of astronomy.

Domino antimagic squares
Alison Marr, Southwestern University

A domino antimagic square of order n is an n by n array formed from a subset of dominoes such that the sums of the rows, columns, and two main diagonals form a set of $2n+2$ distinct, consecutive integers. We will provide examples of domino antimagic squares and discuss some generalizations including domino antimagic rectangles. The talk will conclude with some option questions to explore.

Ludus Regularis–a game from the 10th century
Carla Cardoso, University of Lisbon

Ludus Regularis Seu Clericalis, the Clergy Game, is a board game with dice, invented in the 10th century by Bishop Wibold of Cambrai (France), with the goal of discouraging the involvement of clerics in gambling, and make them experience the pleasures of the practice of a game with a virtuous theme.

Hunting the algorithm (Part2)
Guido Ramellini, MMACA

Based on a simple challenge from the MMACA, three members of the museum set out to "make a point" of the challenge. From here we were looking for a pattern or an algorithm to generalise this beautiful problem, as well as enriching the challenge more and more, turning it upside down. You can never be too adult to play like a child.

FUNDAPROMAT: Spreading the Joy of Mathematics
Jeanette Shakalli, Panamanian Foundation for the Promotion of Mathematics

The Panamanian Foundation for the Promotion of Mathematics (FUNDAPROMAT) is a private non-profit Foundation whose mission is to change the world's perception so that one and all can experience mathematics as accessible, relevant and inherently joyous. In this short talk, Dr. Shakalli will briefly describe what we do in FUNDAPROMAT, how we do it and why we do it. Dr. Shakalli will also share the goals that we have accomplished in FUNDAPROMAT's three years of existence as well as some of our exciting future plans.

Calendars: proposals and ideas
Fernando Blasco

In this talk we focus the attention on a topic that was several times used by Martin Gardner. On it we'll remind Lewis Carroll Method for calculating the day of the week and John Conway's Doomsday algorithm. We'll also make a proposal of an easy calendar and we'll show an easy perpetual calendar model based on Lewis Carroll method.

Matinée Magic Show
Adrien Lochon, France
Colin Wright, Australia
Katerina L'dokova, Belarus
Sergio Belmonte, Spain
Robert Vallin, USA
Circo Matemático, Portugal

During the course of an hour and a half we aim to entertain you with the intellectual extravaganza and nuggets of performance act. Hosted by the Circo Matemático we'll have various show people from all over the globe each with their unique acts and skills.

Love, Mathematics and other portents (in Portuguese), by Jorge Buescu
Carlota Simões, University of COimbra

Mathematics is incredibly fun, of course: why else should we be here? But it is also astonishingly rich in stories, from the curious to the hilarious, from the epic to the outright tragic. This book is a collection of stories about Maths: from the Pope-mathematician to the fabulous Bernoulli dinasty, from the mathematical analogue of "love and a hut" to the tragedy of the Fields medal in the feminine.

Above all, this book is a love letter from a mathematician to its beloved science.